science for global insight

Risk-adjusted Decision Making for Sustainable Development

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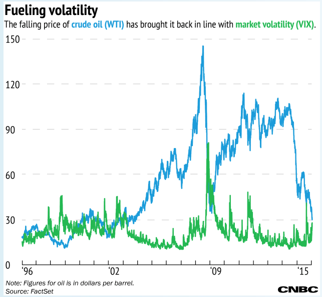
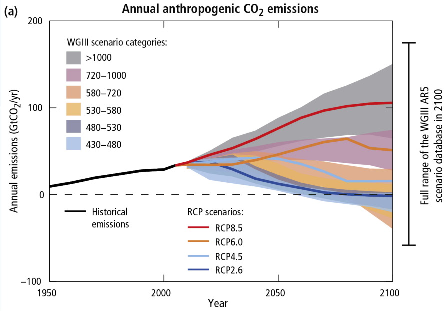
Modeling to inform transitions to sustainability: Four challenges



VOLATILITY



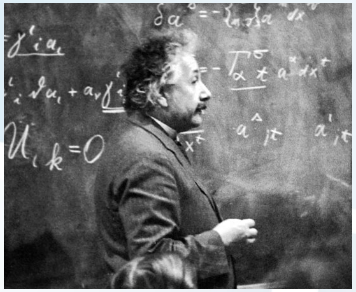
UNCERTAINTY

COMPLEXITY

AMBIGUITY

Notion "VUCA" was introduced by the US Army College to describe the world as resulting from the end of the Cold War

VUCA-world challenges require VUCA-powerful methods to derive effective and efficient solutions



> *"How can it be that mathematics, being after all a product of human thought, which is independent of experience, is so admirably appropriate to the objects of reality?"*
>
> Albert Einstein, 1921

Optimization problem under chance constraints – general formulation

$$\text{Minimize } \; f(x)$$

$$h_j(x) \geq 0 \quad j = 1, \dots, k$$

$$\text{Prob}[g_i(x, \omega) \leq 0] \geq \; q_i \quad i = i, \dots, m, \quad \omega \in \Omega$$

x is a vector of first-stage (strategic) decision variables
f is an objective function
h_i are functions representing deterministic constraints
g_i are functions defining probabilistic chance constraints
q_i are specified target reliabilities
ω is a vector of random variables that represent uncertain parameters
Ω is a set of all possible values of ω

Application to a Water-Energy-Food nexus management problem

Application: Water-Energy-Food nexus

$$\text{Minimize } \; f(x)$$

$$h_j(x) \geq 0 \quad j = 1, \dots, k$$

$$\text{Prob}[g_i(x, \omega) \leq 0] \geq \; q_i \quad i = i, \dots, m, \quad \omega \in \Omega$$

- Focus: A region consisting of sub-regions, each producing coal and growing crops – both require water which is scarce. Demands for coal and crops are given
- Objective function and constraints are linear functions of decision variables
- Major decision variables are the amounts of production of coal by different technologies and the amounts of crops produced in each sub-region -- total **# of sub-regions X number of crops X number of coal technologies** variables
- Additional decision variables: Amounts of coal and crops transported across regions
- Deterministic constraints describe food and energy security
- Probabilistic constraints describe the availability of water
- Uncertainty is the water supply

Solution method – equivalent optimization problem with a penalty term

$$\text{Minimize } \mathrm{E}[F(x,\omega)]$$

$$F(x,\omega) = f(x) - \sum_{i=1}^{m} \alpha_i \, y_i(x,\omega)$$

$$h_j(x) \geq 0 \quad j = 1, \dots, k$$

$$y_i(x,\omega) = \min\{0, -g_i(x,\omega)\}$$

- If the water constraint is satisfied (g_i<0), the penalty term is zero; if it is not satisfied, the unsatisfied water requirement is penalized
- **Target probabilities q_i translate into penalty coefficients alpha_i!!!**

Source: Ermoliev and Wets (1988, pp. 59–61)

Interpretation: two stages

- First stage – "strategic" decisions – taken and implemented before the uncertainty is realized – e.g., installment of water saving technologies

$$\text{Minimize } \mathrm{E}[F(x,\omega)]$$

$$F(x,\omega) = f(x) - \sum_{i=1}^{m} \alpha_i \, y_i(x,\omega)$$

$$h_j(x) \geq 0 \quad j = 1, \ldots, k$$

- Second stage – "adaptive" decisions – taken and implemented after the uncertainty is realized – e.g., "importation" of water from outside the region or reduction of the household consumption

$$y_i(x,\omega) = \min\{0, -g_i(x,\omega)\}$$

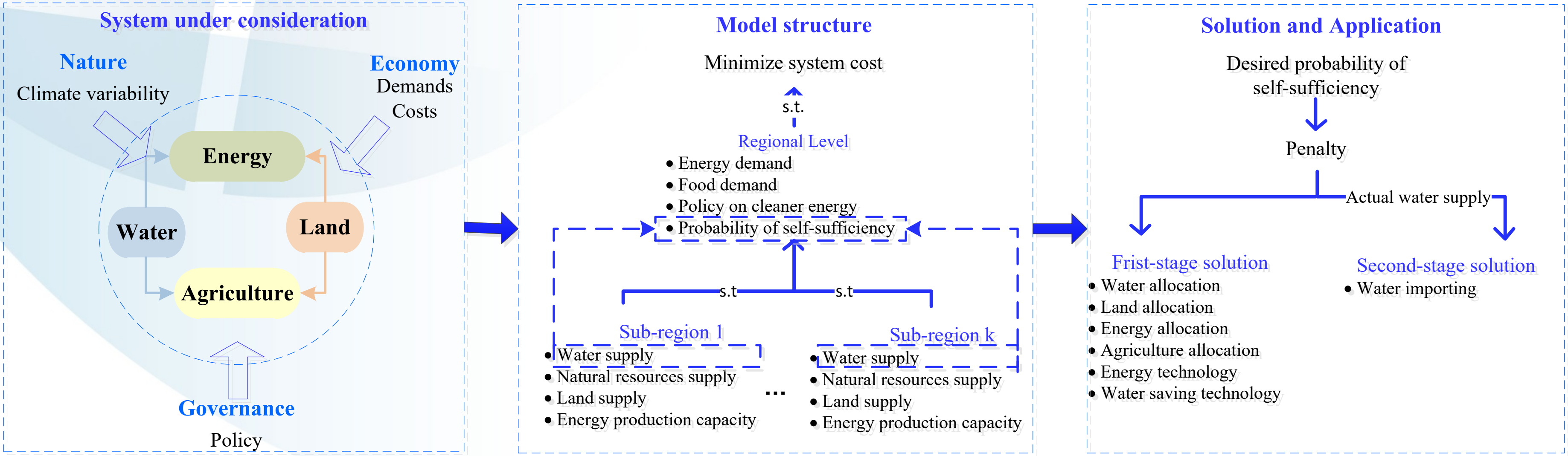
Practical approach to compute the expected value

$$F(x,\omega) \approx f(x) - \sum_{i=1}^{m} \alpha_i \sum_{s=1}^{S} p_s \min\{0, -g_i(x,\omega_s)\}$$

- The expected value of the penalty term is replaced by the sample mean
- Sample is based on observations in the past and/or predictions for future

Source: Ermoliev and Wets (1988)

Model outline



System under consideration

Nature
Climate variability

Economy
Demands
Costs

Energy
Water
Land
Agriculture

Governance
Policy

Model structure

Minimize system cost

s.t.

Regional Level
- Energy demand
- Food demand
- Policy on cleaner energy
- Probability of self-sufficiency

s.t

s.t

Sub-region 1
- Water supply
- Natural resources supply
- Land supply
- Energy production capacity

…

Sub-region k
- Water supply
- Natural resources supply
- Land supply
- Energy production capacity

Solution and Application

Desired probability of
self-sufficiency

Penalty

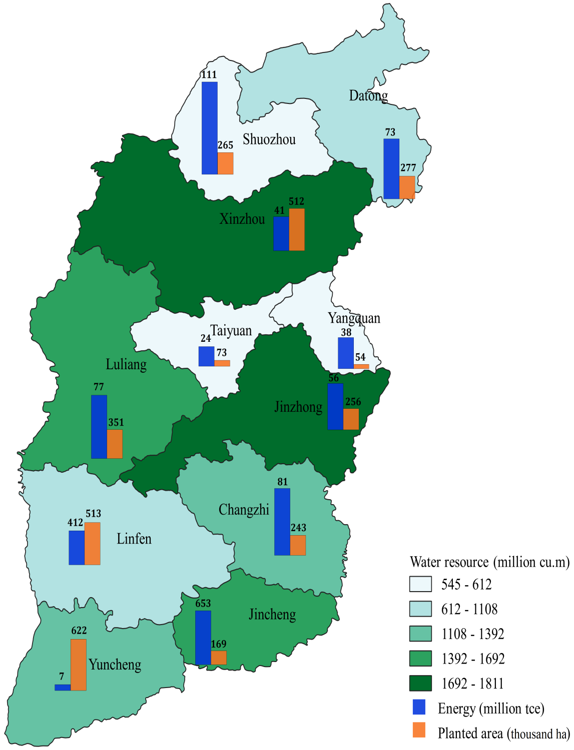
Actual water supply

Frist-stage solution
- Water allocation
- Land allocation
- Energy allocation
- Agriculture allocation
- Energy technology
- Water saving technology

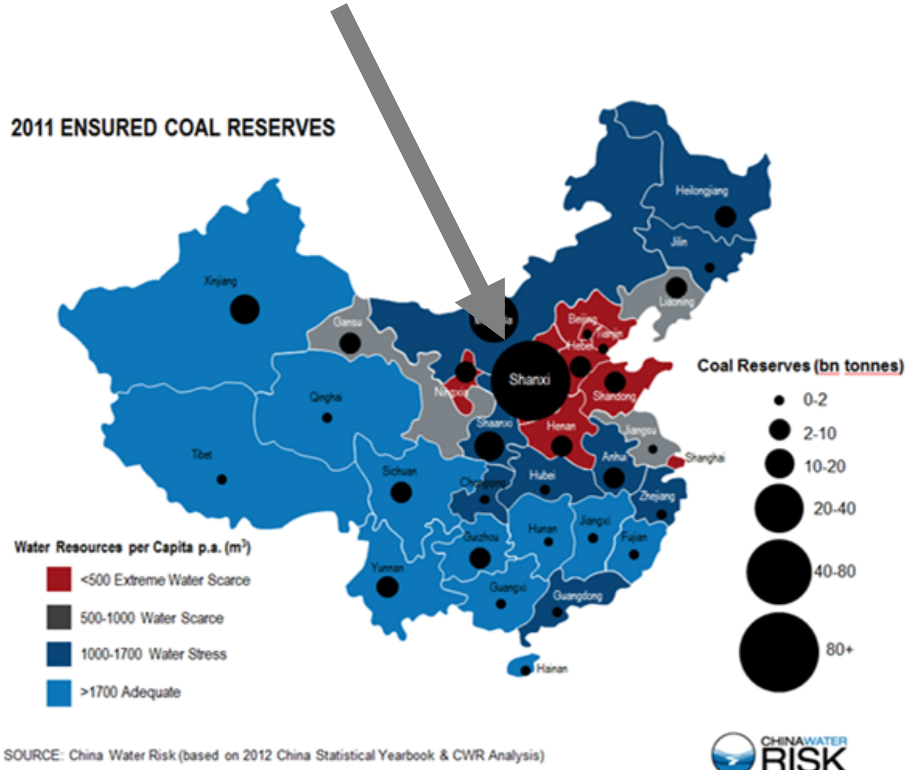
Second-stage solution
- Water importing

Source (here and in the next slides): Gao et al (2021)

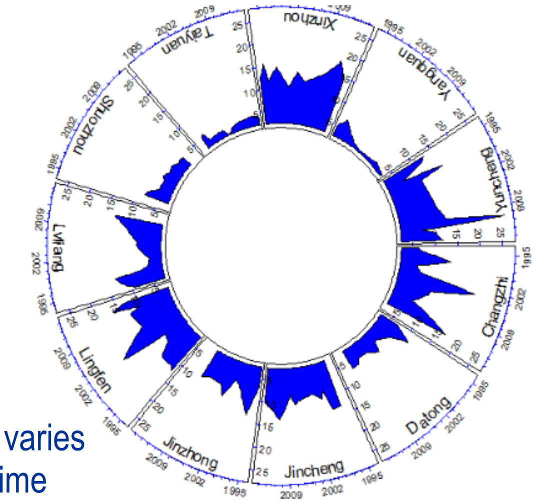
Case study area: Shanxi Province, China



Water, energy and agriculture across Shanxi prefecture

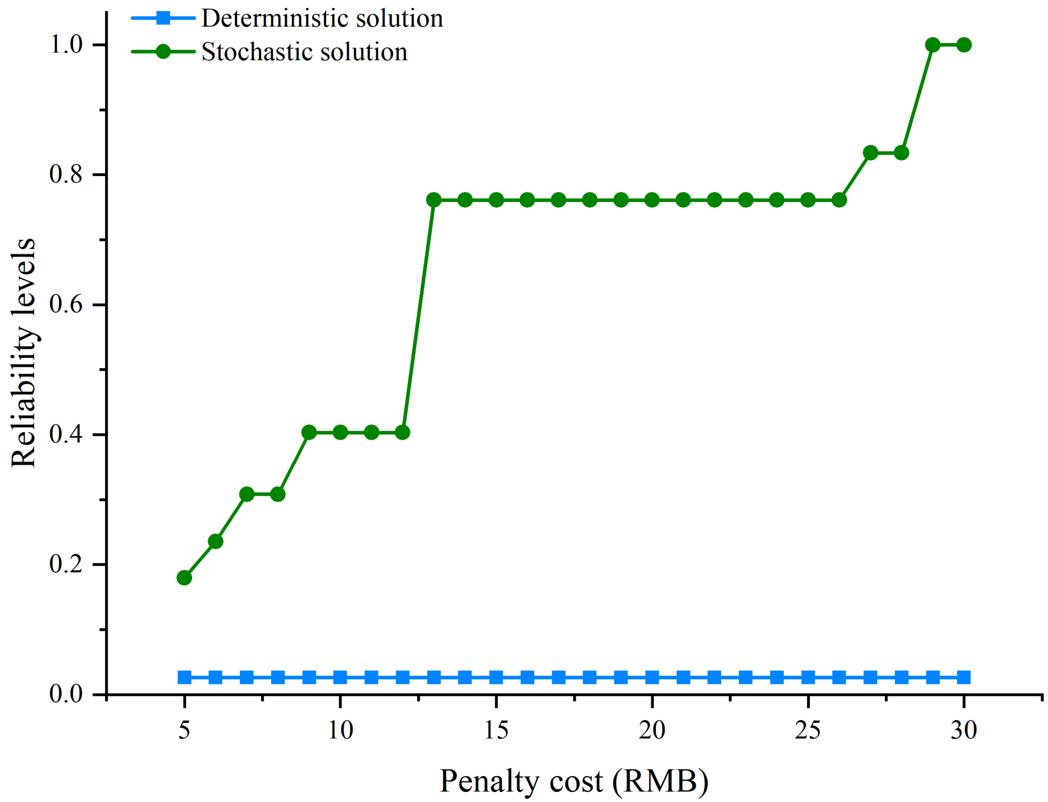


SOURCE: China Water Risk (based on 2012 China Statistical Yearbook & CWR Analysis)



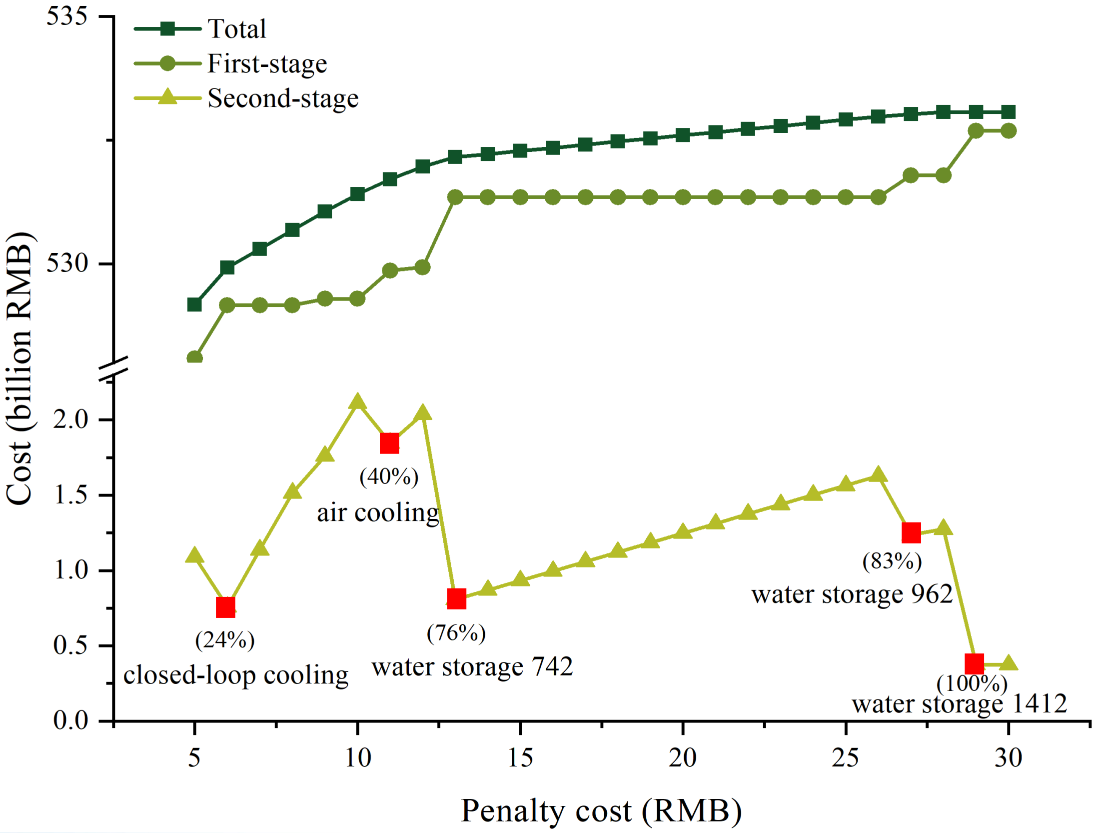
Water availability in Shanxi varies across prefectures and time

Reliability-penalty relation



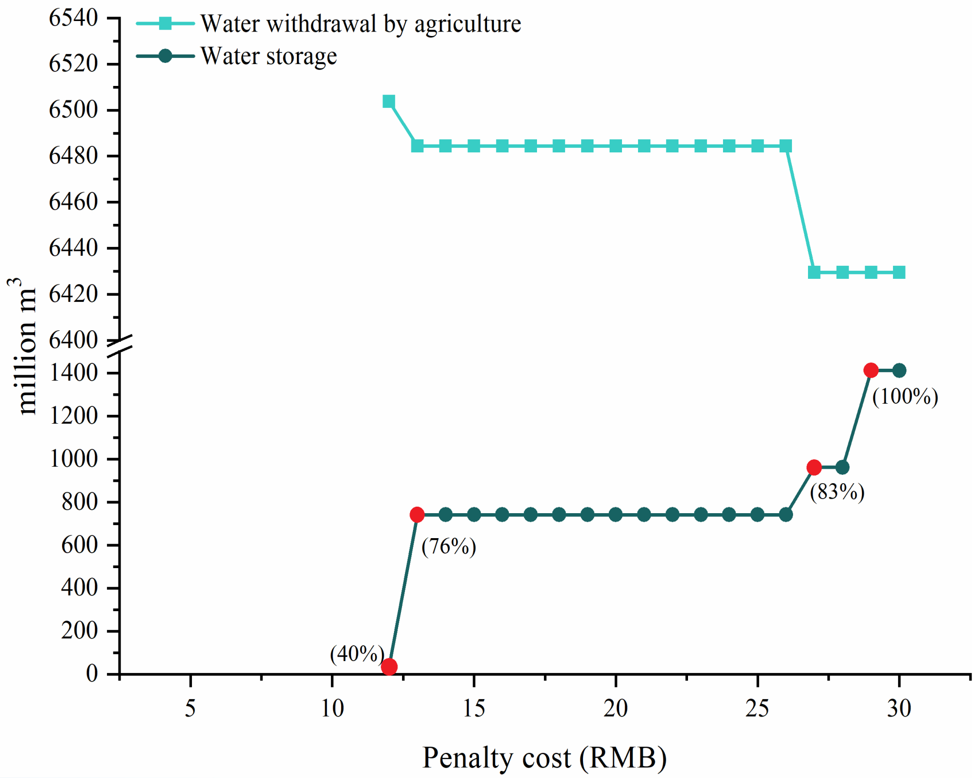
Reliability levels under different penalties, and comparison of the stochastic and deterministic solutions

Solution costs



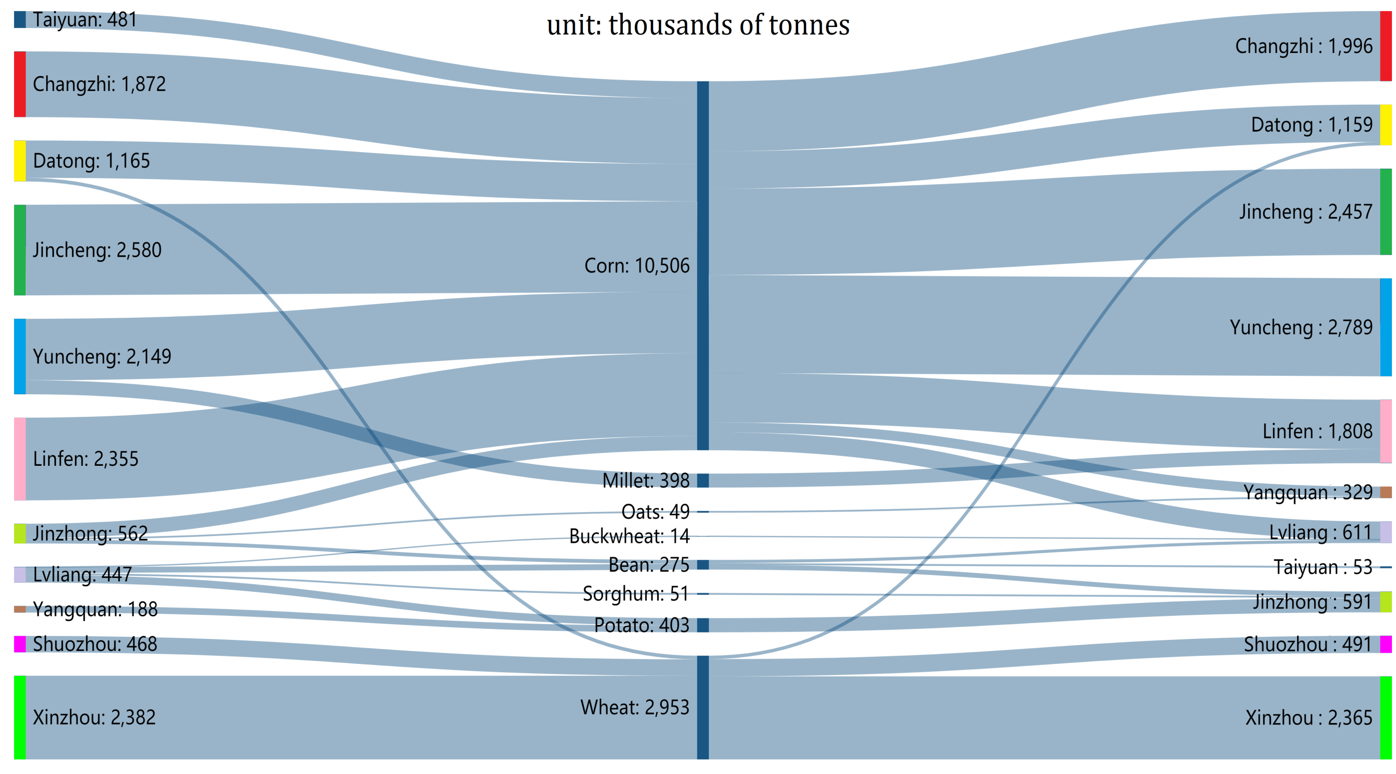
Total, first-stage, and the expected value of second-stage costs of the stochastic solution at different penalty levels. The numbers 742, 962, and 1412 represent optimal volumes of water storage, in million m^3. Percentages in parenthesis indicate the reliability levels that can be achieved at the corresponding penalty level due to the deployment of the indicated technologies in the first stage.

Water saving solutions



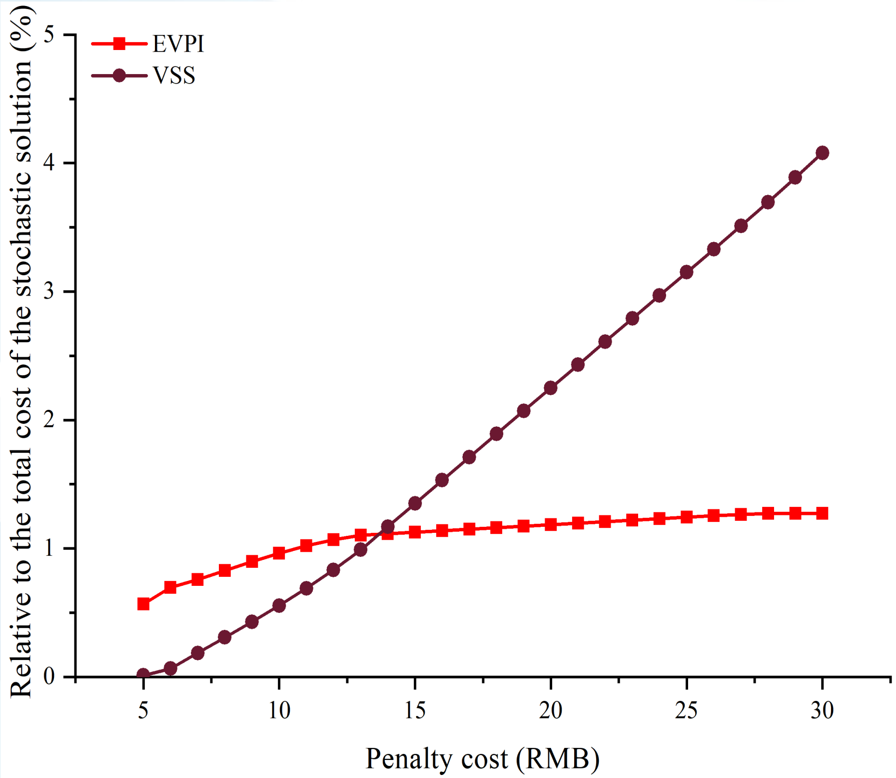
Amount of water storage at the second stage and water withdrawal by crops throughout the entire province at different penalty levels. At penalty levels lower than 12 RMB per ton, water storage is zero. Percentages in parenthesis indicate the reliability levels that can be achieved at the corresponding penalty level by establishing water storage at the indicated capacity in the first stage.

Optimal allocation of production



Optimal crop production in each sub-region under 11 RMB per ton of water penalty (left part of the figure) and 12 RMB per ton of water penalty (right part of the figure).

What is the benefit of information?



$$\mathrm{VSS} = \mathrm{E}F\left(x^{\mathrm{E}\omega}, \omega\right) - \mathrm{E}F(x^*, \omega)$$

$$\mathrm{EVPI} = \mathrm{E}F(x^*, \omega) - \mathrm{E}f(x^\omega, \omega)$$

$x^*$is the first-stage solution in the two-stage stochastic optimization problem

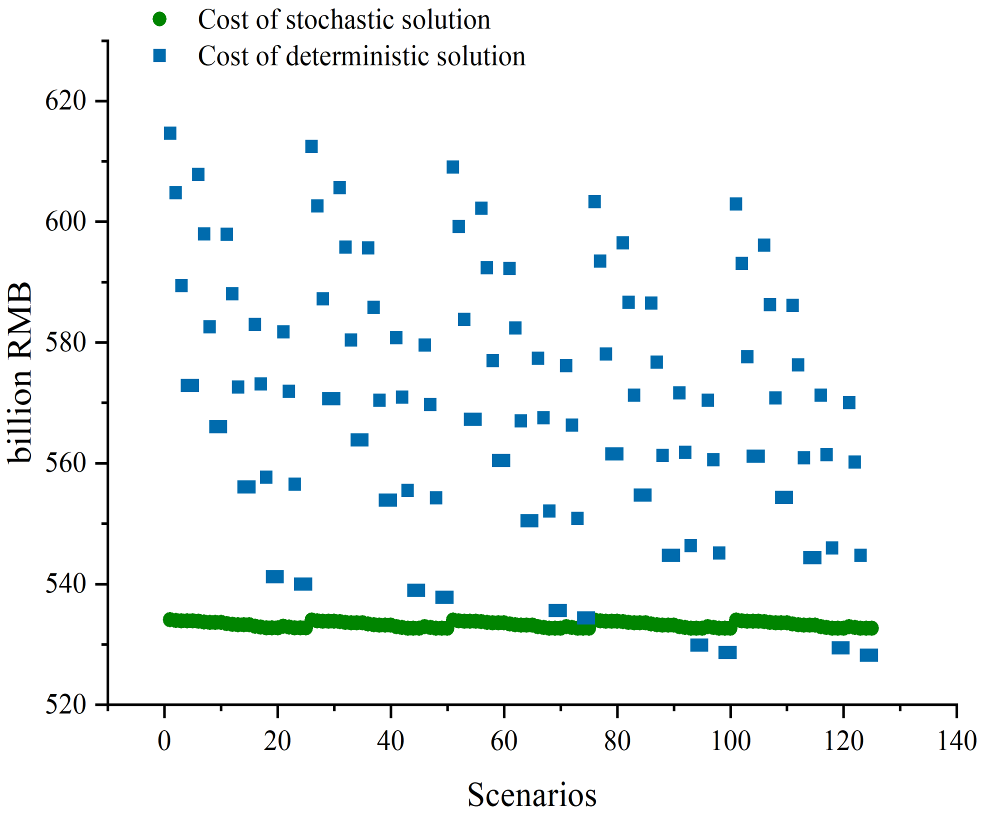
x^ω is the (first-stage) solution in the deterministic optimization problem

$$\text{Minimize } f(x) - \sum_{i=1}^{m} \alpha_i \min\{0, -g_i(x, \omega)\}$$

$$h_j(x) \geq 0 \quad j = 1, \dots, k$$

The expected value of perfect information (EVPI) and the value of the stochastic solution (VSS) relative to the total cost of the stochastic solution under different penalties.

Robustness of the stochastic solution



For each water availability scenario, the total costs of the deterministic and stochastic solutions for a penalty of 30 RMB per ton of water.

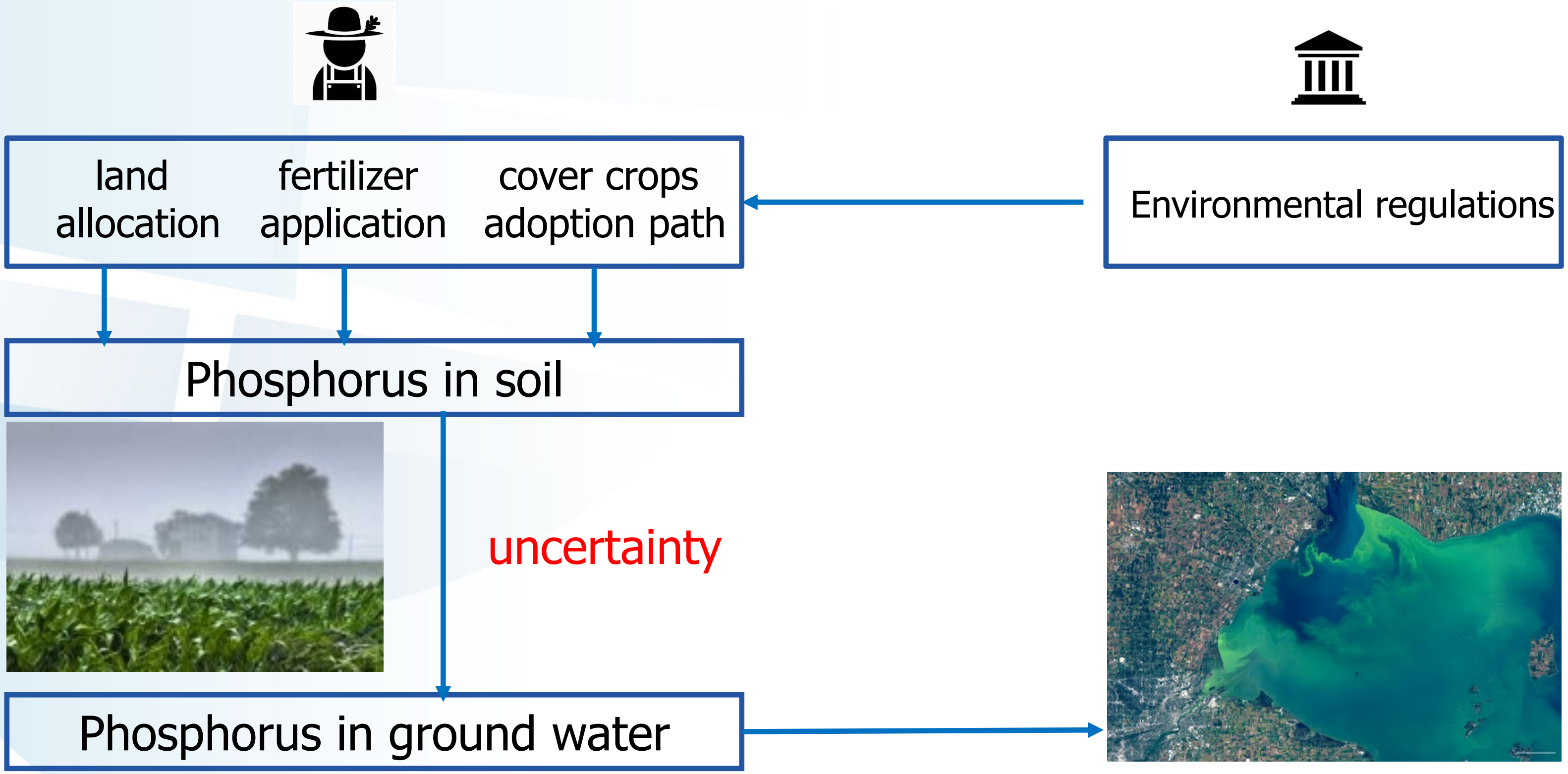
Application to Pollution Control in Surface Waters

Pollution control: Eutrophication in surface waters

Lake Erie, North America
Source: New York Times, 2017

Baltic Sea
Source: The Guardian, 2020

Fertilizers and pollution control



land allocation

fertilizer application

cover crops adoption path

Environmental regulations

Phosphorus in soil

uncertainty

Phosphorus in ground water

Source: Wildemeersch et al (2019)

Chance constraint problem

- Dynamics of the Phosphorus cycle taken into account
- Expected profit is maximized over Phosphorus application, crop allocation and cover crops
- Uncertain emissions into the lake need to be limited with high reliability

$$\max_{\delta_i,\theta_i,F_i} \quad \sum_{t=1}^{\infty} \beta^t \sum_i \boxed{\delta_i}(t)\,\mathbb{E}\left[\pi_i\left(\boxed{\theta_i}(t),\boxed{F_i}(t),\omega\right)\right]$$

Profit function as a function of crop yields

$$\text{s.t.} \quad \text{state equations}$$

$$\boxed{\begin{aligned} &\mathbb{P}\left[E^{\mathrm{sa}}(t,\omega) \leq \eta^{\mathrm{sa}}\right] \geq 1-\epsilon^{sa} \\ &\mathbb{P}\left[E^{\mathrm{ss}}(t,\omega) \leq \eta^{\mathrm{ss}}\right] \geq 1-\epsilon^{ss} \end{aligned}}$$

Phosphorus emissions per unit area

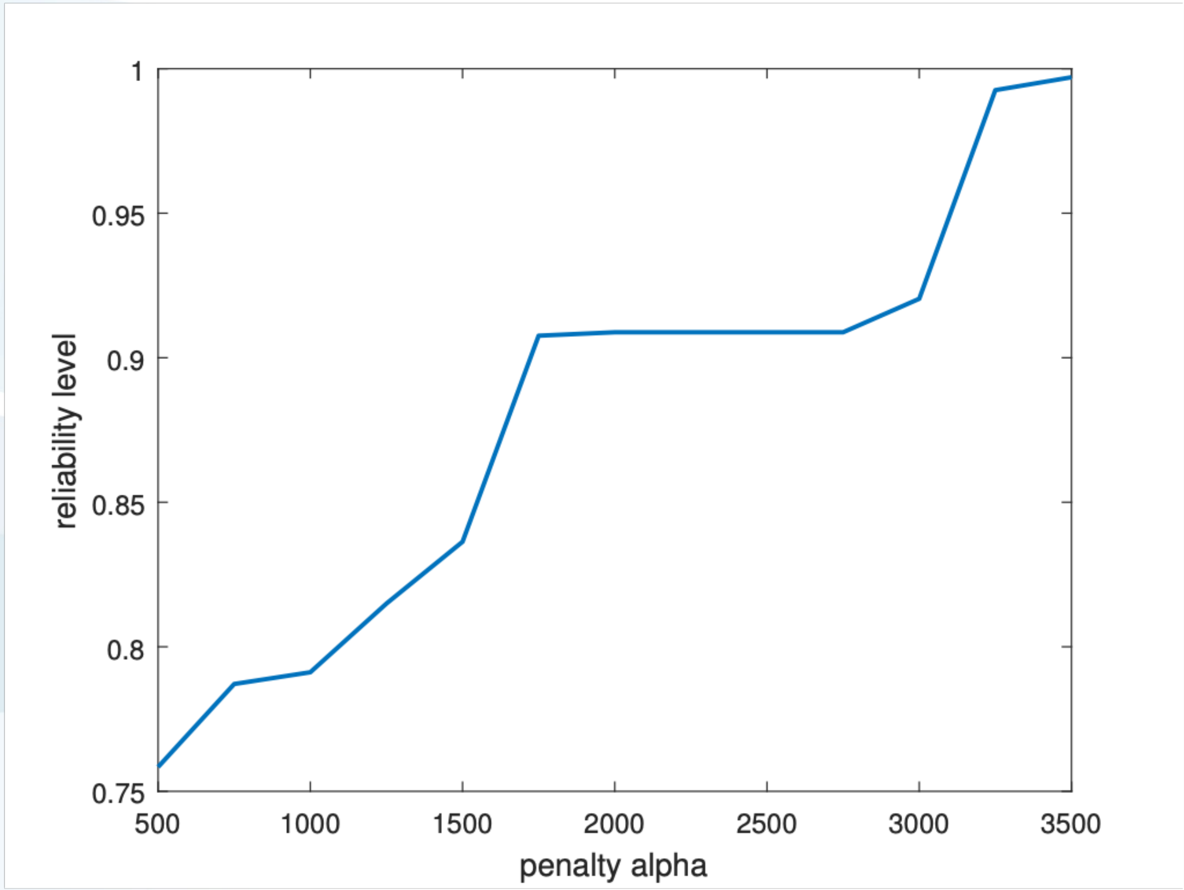
probabilistic constraints

Reformulation as two-stage problem with recourse

Fertilizer application, crop allocation, and farming practices (strategic decisions) need to be chosen wisely to avoid paying too large penalties (adaptive decisions)

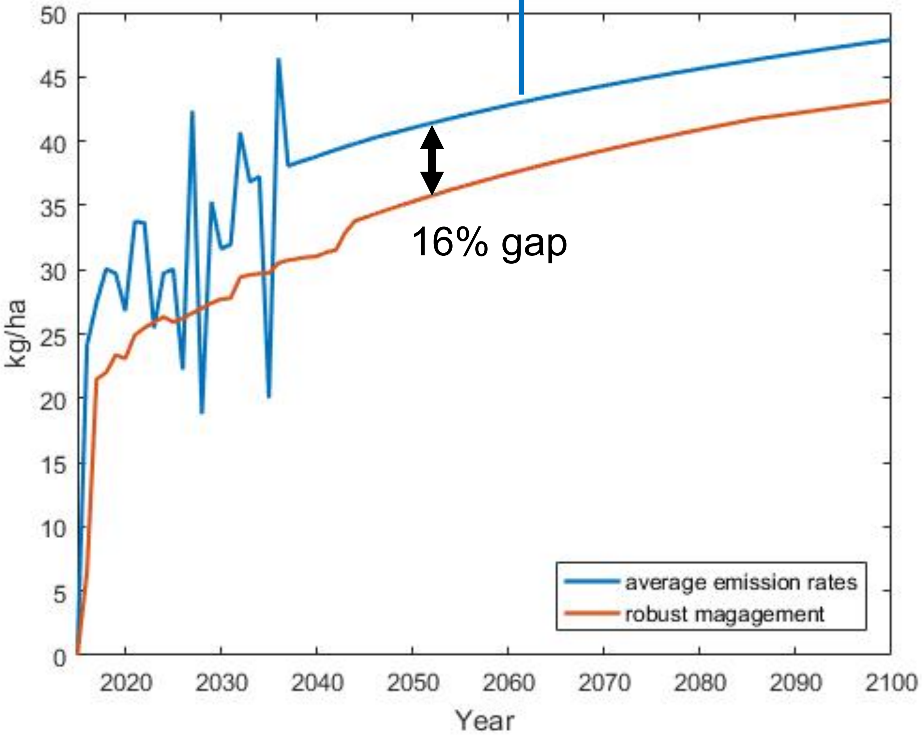
$$\begin{aligned} \max_{\delta_i,\theta_i,F_i} \quad & \sum_{t=1}^{\infty} \beta^t \left(\overset{\text{Profit}}{\boxed{\sum_i \delta_i(t)\mathbb{E}\left[\pi_i\Big(\theta_i(t), F_i(t), \omega\Big)\right]}} - \overset{\text{Environmental cost}}{\boxed{\alpha\Big(\mathbb{E}\big[\max\{0, E^{\text{sa}}(t,\omega) - \eta^{\text{sa}}\} + \max\{0, E^{\text{ss}}(t,\omega) - \eta^{\text{ss}}\}\big]\Big)}} \right) \\ \text{s.t.} \quad & \text{state equations} \end{aligned}$$

Relationship between risk level and corresponding cost



Robust solution provides significant tighter guidelines for fertilizer application

cause of algal blooms



P fertilizer application rate over time

- Fertilizer application based on mean emission rates has historically led to harmful algal blooms
- Robust fertilizer application rate 16% lower than application based on mean emission rate

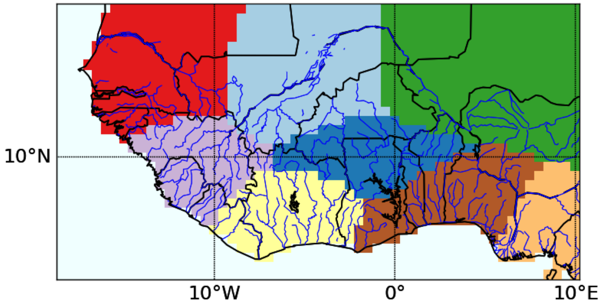
Application to Financing Mechanisms for Sustainable Food Security

Risk management for sustainable food security

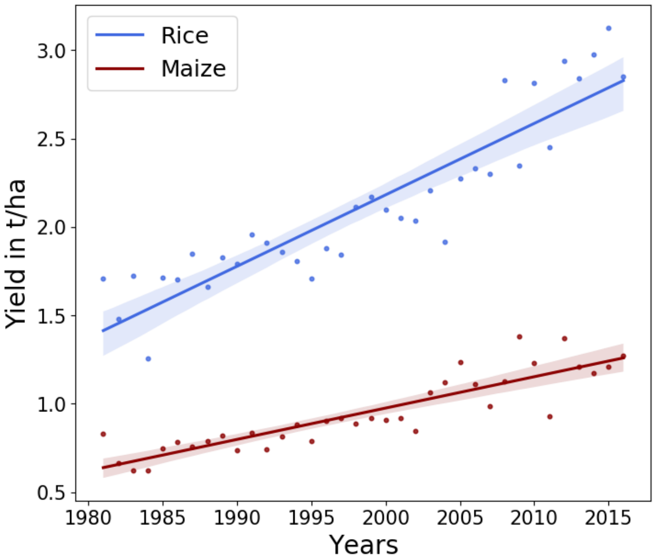
- Population is growing very quickly in West-Africa
- Farmers in the region are faced with extreme weather events
- Stable incomes are necessary to avoid poverty traps and allow for investment in efficient farming technologies.
- Can local food production keep up with population growth?
- Is it possible to provide more stable incomes to local farmers by means of a catastrophe fund?
- How much risk pooling needs to occur to ensure food security and/or solvency of the catastrophe fund?

Modeling food security and financial risk transfer (i)

- Establish dependence structure of crop yields in West-Africa



- Define crop yield projections for the coming 25 years
  - Uncertainty: random vector of yields over different clusters
  - Non-stationary process: expected yield is varying with time



Modeling food security and financial risk transfer (ii)

- SO model that
  - Minimizes cultivation cost
  - Food security needs to be guaranteed with given reliability level
  - Catastrophe fund needs to stay solvent after catastrophic event
  - Crop choice and land allocation (strategic decisions) need to be chosen wisely to avoid importing food at higher price and taking loan (adaptive decisions)

$$\min_{x(t)} \quad \mathbb{E}\left[\sum_{t=t_0}^{T_{\text{fin}}} \sum_k \sum_j \overset{\text{Cultivation cost}}{\boxed{c_j}} \cdot \underset{\text{land allocation}}{\boxed{x_{j,k,t}}}\right]$$

Minimize expected cultivation costs incurred over time, clusters, and crops

$$\text{s.t.} \quad \mathbb{P}\left[\sum_k \sum_j \underset{\text{random yield}}{\boxed{Y_{j,k,t}}} \cdot x_{j,k,t} \cdot a_j \geq A_t\right] \geq \alpha_{\text{F}}$$

Food security constraint depending on random yield and land allocation

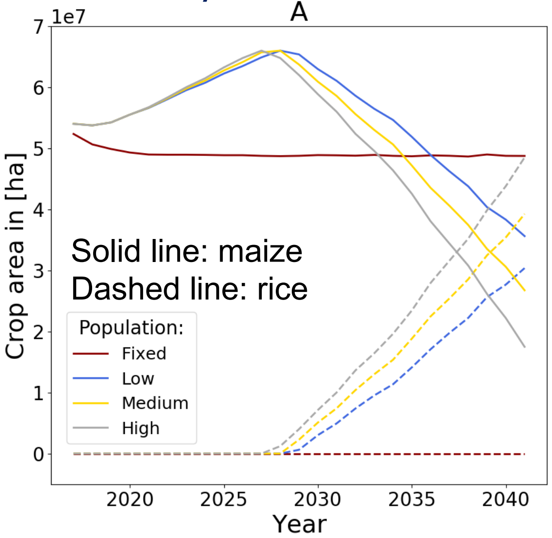
$$\mathbb{P}\left[\underset{\text{Capital in fund after catastrophe}}{\boxed{G_{\text{fin}}}}(x, Y) \geq 0\right] \geq \alpha_{\text{S}}$$

Solvency constraint for the catastrophe fund after a catastrophic event

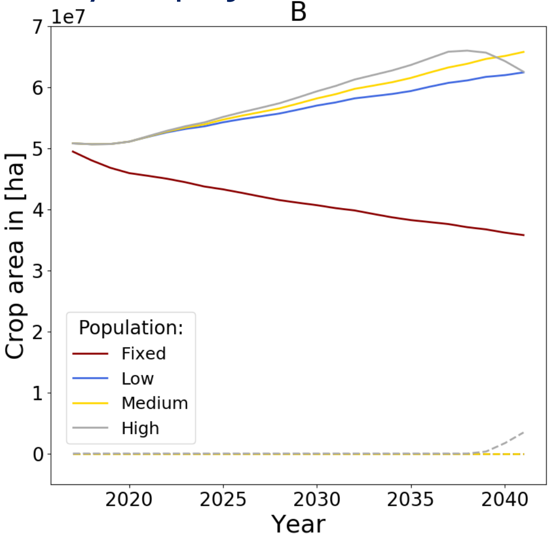
Managing food security and insolvency risk

- We consider different population scenarios and analyze the effects on allocated land
- Food security probability at 95% and solvency probability at 85%
- Food security constraint and solvency constraint not active at the same time
- Maize substituted by rice once limit of arable land is reached
- Currently, arable land is sufficient to meet food demand with high probability, but population growth is a critical factor for sustainable food security in West Africa.

Fixed yield distribution

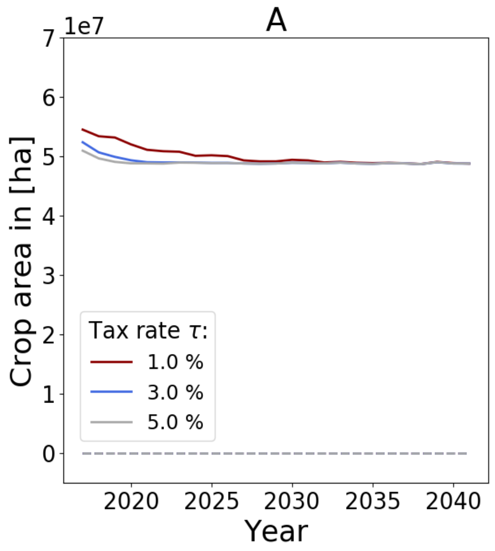


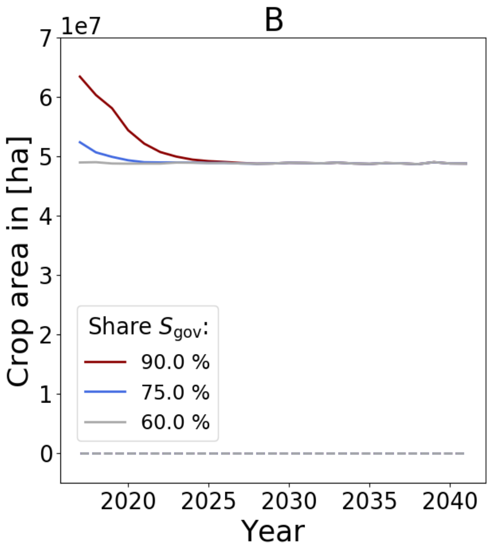
yield projections included

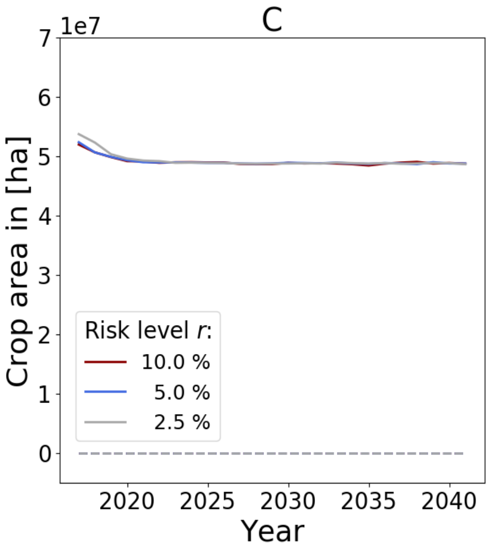


Government interventions to finance catastrophe fund

- We consider three policy levers that can be used by governments to manage food security and stabilize incomes
- Tax rate affects time needed to build up the fund
- Share of guaranteed income requires large initial overproduction
- Increased risk level reduces the expected losses to be refunded to farmers

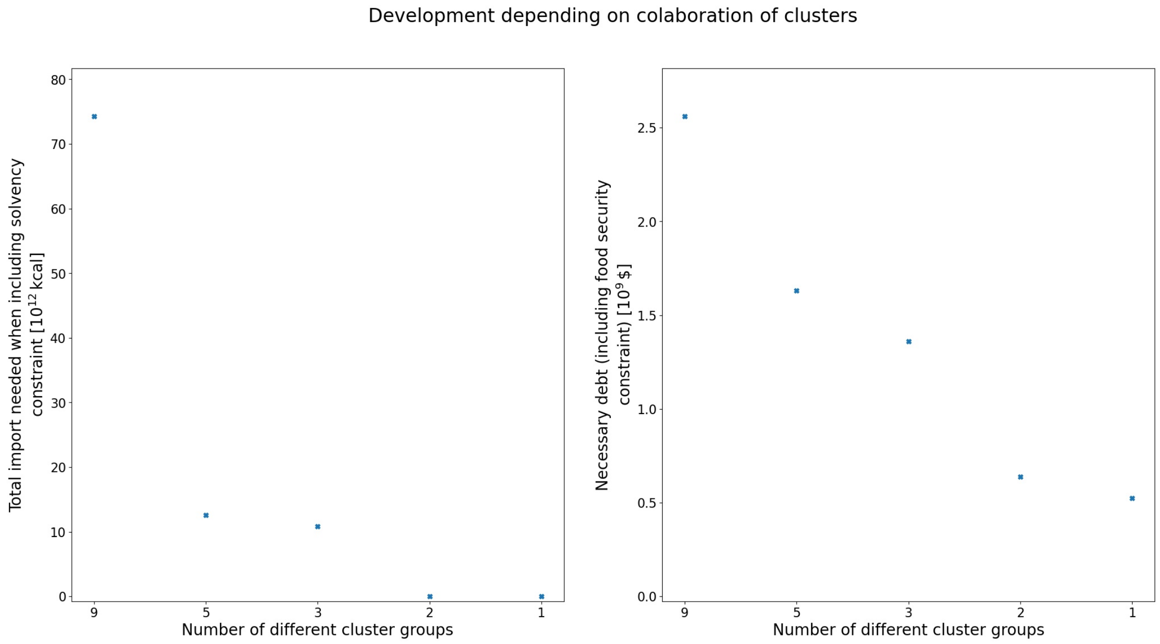

Tax rate


Share of guaranteed income


Insured risk level

Spatial correlation and risk pooling

- We quantify the effect of risk pooling by evaluating the total amount of food imports and the total debt that needs to be sustained for different levels of cooperation.
- Collaboration between different clusters improves the feasibility to achieve food security and solvency of the financing mechanism.
- Limited levels of cooperation result in large benefits for the food security, and cooperation over larger areas is necessary to improve the solvency objective.



Food security probability 99%
Solvency probability 95%
Covered risk level 5%
Tax rate 1%
Percentage of expected income guaranteed 90%